Curves you can trust

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SECUERity

Common tool for attacking isogeny schemes:

KNOWN ENDOMORPHISMS

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Tiny little problem: How to construct curves with **UNKNOWN ENDOMORPHISMS**?

Folklore solution

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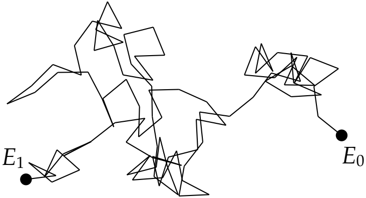
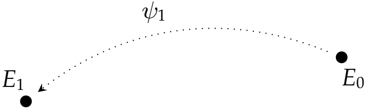
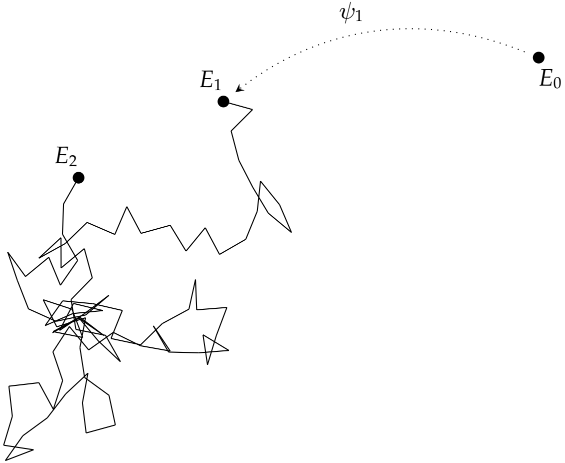
$\Longrightarrow$ **Distribute the trust:**

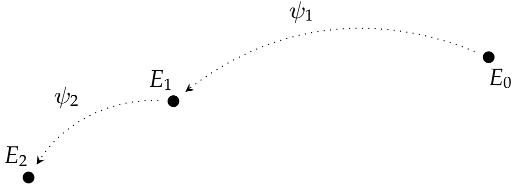
Chain secret random walks generated by different people.
Prevent cheating with a **Proof of Isogeny Knowledge**.

E_0


E_1
E_0


ψ_1
E_1
E_0


ψ_1
E_1
E_0
E_2


ψ_1
E_1
E_0
ψ_2
E_2

Our work

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- Statistically zero-knowledge.
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- Planning to run a ceremony in reality!

Application: Defending SIDH/SIKE in depth

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https://ia.cr/2022/975
https://ia.cr/2022/1026
https://ia.cr/2022/1038
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SIDH squares are (still!) our best friend

$$\begin{tikzcd}
E_0 \arrow[r] \arrow[d] & E_A \arrow[d] \\
E_B \arrow[r] & E_{AB}
\end{tikzcd}$$

Formerly known as “SIDH square” ☠.

SIDH squares are (still!) our best friend

$$\begin{tikzcd}
& E_0 \arrow[dl] \arrow[dr] & \\
E_A \arrow[dr] & & E_B \arrow[dl] \\
& E_{AB} &
\end{tikzcd}$$

Formerly known as “SIDH square” ☠.

Diamonds are our best friend

Formerly known as "SIDH square" ☠. Now rebranded as

isogeny diamond.

Thanks!