

How 2 Sage Dev??

Lorenz Panny

Technische Universität München

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Part 4: If it's broke, fix it!



(Slide from my Isogeny Days 2022 talk.)

Outline

What is SageMath, really?

The type system The three type systems

Getting your hands dirty

It's working! (That means ε % of the work is done.)

Ready? Go!

 Sage rests on Python interfaces to mathematical libraries: They provide most of the <u>fundamental</u> functionality.

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 - ► GMP & MPFR: Arbitrary-precision arithmetic.
 - PARI: Odd finite fields, univariate polynomials, power series, elliptic curves, quadratic forms, number fields, class groups, ...
 - ▶ NTL: **F**_{2ⁿ}, various algorithms for fast arithmetic, ...
 - ► Singular: Multivariate polynomials, Gröbner bases, elimination, ...
 - ► fplll: Lattice algorithms.
 - Linbox, M4RI: Linear algebra.
 - ► GAP: Abstract groups.

The gory internals (1): Example

polmodular

🔁 û

Return the modular polynomial of prime level L in variables x and y for the modular function specified by inv. If inv is 0 (the default), use the modular j function, if inv is 1 use the Weberf function, and if inv is 5 use $\gamma_2 = sqrt[3]\{j\}$. See polclass for the full list of invariants. If x is given as Mod(j, g) or an element j of a finite field (as a \bot FFLT), then return the modular polynomial of level L evaluated at j. If j is from a finite field and derivs is non-zero, then return a triple where the last two elements are the first and second derivatives of the modular polynomial evaluated at j.

```
? polmodular(3)
  1 = x^4 + (-y^3 + 2232^*y^2 - 1069956^*y + 36864000)^*x^3 + \dots
  ? polmodular(7, 1, , 'J)
  \frac{1}{2} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{4} + \frac{1}{4} + \frac{1}{7} + \frac{1}
  ? polmodular(7, 5, 7*ffgen(19)^0, 'j)
  %3 = j^8 + 4*j^7 + 4*j^6 + 8*j^5 + j^4 + 12*j^2 + 18*j + 18
  ? polmodular(7, 5, Mod(7,19), 'j)
  $4 = Mod(1, 19)*i^8 + Mod(4, 19)*i^7 + Mod(4, 19)*i^6 + ...
  ? u = ffgen(5)^0: T = polmodular(3.0..'i)*u:
  ? polmodular(3, 0, u, j,1)
  \%6 = [j^4 + 3*j^2 + 4*j + 1, 3*j^2 + 2*j + 4, 3*j^3 + 4*j^2 + 4*j + 2]
  ? subst(T.x.u)
\%7 = i^4 + 3*i^2 + 4*i + 1
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The library syntax is GEN polmodular(long L, long inv, GEN x = NULL, long y = -1, long derivs) where y is a variable number.

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```
sage: from sage.libs.pari import pari # imported by default
sage: pari.polmodular(2)
x^3 + (-y^2 + 1488*y - 162000)*x^2
+ (1488*y^2 + 40773375*y + 8748000000)*x
+ (y^3 - 162000*y^2 + 8748000000*y - 15746400000000)
```

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sage: type(pari.polmodular(2))
<class 'cypari2.gen.Gen'>
```

- src/sage/rings/finite_rings/...
- src/sage/rings/polynomials/...
- src/sage/algebras/quatalg/...
- src/sage/schemes/elliptic_curves/...
- src/sage/rings/number_field/...
- src/sage/quadratic_forms/...

► On top of that: sagelib, all the "native" Sage (Python) code.

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sage: E = EllipticCurve(j=42)
sage: E.isogeny?

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Signature:
E.isogeny(
    kernel,
    codomain=None,
    degree=None.
    model=None.
    check=True,
    algorithm=None.
)
Docstring:
    Return an elliptic-curve isogeny from this elliptic curve.
# ...
Init docstring: Initialize self. See help(type(self)) for accurate signature.
File:
                 ~sage/src/sage/schemes/elliptic_curves/ell_field.py
Type:
                 method
```

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sage: E = EllipticCurve(i=42)
sage: E.isogeny??
Signature:
E.isogenv(
# ...
# ...
 if algorithm == "velusort":
    from sage.schemes.elliptic curves.hom velusart import EllipticCurveHom velusart
    return EllipticCurveHom_velusqrt(self, kernel, codomain=codomain, model=model)
 if algorithm == "factored":
    from sage.schemes.elliptic curves.hom composite import EllipticCurveHom composite
    return EllipticCurveHom_composite(self, kernel, codomain=codomain, model=model)
 try:
    return EllipticCurveIsogenv(self, kernel, codomain, degree, model, check=check)
  except AttributeError as e:
    raise RuntimeError("Unable to construct isogeny: %s" % e)
           ~sage/src/sage/schemes/elliptic curves/ell field.pv
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```
sage: E = EllipticCurve(j=42)
sage: E.isogeny???
Cell In[1], line 2
E.isogeny???
^
SyntaxError: invalid syntax
```



```
sage: E = EllipticCurve(GF(419^2), [1,0])
sage: pi = E.frobenius_isogeny()
sage: type(pi)
<class 'sage.schemes.elliptic_curves.hom_frobenius.EllipticCurveHom_frobenius'>
```

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```

```
sage: import_statements(order_from_multiple)
# ...
NameError: name 'order_from_multiple' is not defined
sage: import_statements('order_from_multiple')
from sage.groups.generic import order_from_multiple
```

Common Sage vs. Python pitfalls

Sage	Python
(nothing)	from sage.somewhere import Something
^	**
~ ^	^
42	Integer(42)
R. <x> = Thing</x>	R,x = Thing.objgen()

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- ► Inheritance: Copy code and data from an ancestor class. For example, EllipticCurve_finite_field inherits from EllipticCurve_field.

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- ► Lots of conventions for how these "things" behave.
 - ▶ Particular SageObjects should not override (say) .__add__().
 - Instead, "things" first defer to the coercion system for type checks and conversions, which *then* calls your (say) ._add_().

```
# from sage.quadratic_forms.bqf_class_group.BQFClassGroup_element
def _add_(self, other):
    r""" ... """
    F = self__form * other._form
    return BQFClassGroup_element(F, parent=self.parent())
```

Previous slide:

Each kind of "thing" is split into three seemingly independent types: Elements, Morphisms, Parents.

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 - As far as I can tell, this is still not widely used.
 - General advice: Mimic existing, similar code.

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- 2. Wait for someone else to put it *into* Sage.

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- 3. Something's unreasonably slow? Report it.

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- 4. Unless satisfied, go back to Step 1 and <u>iterate</u>.

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The next best thing™:

 After debugging, <u>comment out</u> asserts (instead of deleting them). It aids later debugging sessions, and it helps convey your intentions behind the code.

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- $\rightsquigarrow\,$ Make sure your Sage patch adheres to the rules.

https://doc.sagemath.org/html/en/developer/coding_basics.html#python-code-style

Example from EllipticCurveHom_velusqrt (part 1/4):

```
r"""
This class implements separable odd-degree isogenies of elliptic
curves over finite fields using the square-root Vélu algorithm.
The complexity is `\tilde O(\sqrt{\ell})` base-field operations,
where `\ell` is the degree.
REFERENCES: [BDLS2020]_
INPUT:
- ``E`` -- an elliptic curve over a finite field
- ``P`` -- a point on `E` of odd order `\geq 9`
- ``codomain`` -- codomain elliptic curve (optional)
- ``model`` -- sring (optional); input to
imeth:`~sage.schemes.elliptic_curves.ell_field.compute_model`
- ``Q`` -- a point on `E` outside `\langle P\rangle`, or ``None``
# ...
```

...

Example from EllipticCurveHom_velusqrt (part 2/4):

```
r"""
#
EXAMPLES::
    sage: from sage.schemes.elliptic_curves.hom_velusqrt import
                                                   EllipticCurveHom_velusqrt
    sage: F_{,<t>} = GF(10009^{3})
    sage: E = EllipticCurve(F. [t.t])
    sage: K = E(2154*t^2 + 5711*t + 2899, 7340*t^2 + 4653*t + 6935)
    sage: phi = EllipticCurveHom velusort(E, K); phi
    Elliptic-curve isogeny (using square-root Vélu) of degree 601:
      From: Elliptic Curve defined by y^2 = x^3 + t + t
              over Finite Field in t of size 10009^3
      To:
             Elliptic Curve defined by v^2 = x^3 + (263 \pm t^2 + 3173 \pm t + 4759) \pm x
              + (3898*t^2+6111*t+9443) over Finite Field in t of size 10009^3
    sage: phi(K)
    (0:1:0)
    sage: P = E(2, 3163 \times t^2 + 7293 \times t + 5999)
    sage: phi(P)
    (6085*t^2 + 855*t + 8720 : 8078*t^2 + 9889*t + 6030 : 1)
    sage: 0 = E(6, 5575 \times t^2 + 6607 \times t + 9991)
    sage: phi(0)
    (626*t^2 + 9749*t + 1291 : 5931*t^2 + 8549*t + 3111 : 1)
    sage: phi(P + 0)
    (983*t^2 + 4894*t + 4072 : 5047*t^2 + 9325*t + 336 : 1)
    sage: phi(P) + phi(0)
    (983 \pm 1^{2} \pm 4894 \pm 1 \pm 4072 \pm 5047 \pm 1^{2} \pm 9325 \pm 1 \pm 336 \pm 1)
```

Example from EllipticCurveHom_velusqrt (part 3/4):

```
r"""
# ...
TESTS:
Check on a random example that the isogenv is a well-defined
group homomorphism with the correct kernel::
    sage: from sage.schemes.elliptic curves.hom velusort import
                                                random example for testing
    sage: E, K = _random_example_for_testing()
    sage: phi = EllipticCurveHom velusart(E, K)
    sage: not phi(K)
    True
    sage: not phi(randrange(2^99) * K)
    True
    sage: P = E.random_point()
    sage: phi(P) in phi.codomain()
    True
    sage: 0 = E.random point()
    sage: phi(0) in phi.codomain()
    True
    sage: phi(P + 0) == phi(P) + phi(0)
    True
# ...
```

Example from EllipticCurveHom_velusqrt (part 4/4):

```
r"""
# ...
Check that the isogeny preserves the field of definition ::
    sage: Sequence(K).universe() == phi.domain().base_field()
    True
    sage: phi.codomain().base field() == phi.domain().base field()
    True
Check that the isogenv affects the Weil pairing in the correct way::
    sage: m = lcm(P.order(), 0.order())
    sage: e1 = P.weil_pairing(0, m)
    sage: e^2 = phi(P), weil pairing(phi(0), m)
    sage: e2 == e1^phi.degree()
    True
Check that the isogeny matches (up to isomorphism) the one from
:class:`~sage.schemes.elliptic_curves.ell_curve_isogeny.EllipticCurveIsogeny`::
    sage: psi = EllipticCurveIsogenv(E, K)
    sage: check = lambda iso: all(iso(psi(0)) == phi(0) for 0 in E.gens())
    sage: any(map(check, psi.codomain().isomorphisms(phi.codomain())))
    True
... SEEALSO::
    :class: `~sage.schemes.elliptic curves.ell curve isogenv.EllipticCurveIsogenv`
.....
```

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scaling_factor()

Return the Weierstrass scaling factor associated to this elliptic-curve morphism.

The scaling factor is the constant u (in the base field) such that $\varphi^*\omega_2 = u\omega_1$, where $\varphi: E_1 \to E_2$ is this morphism and ω_i are the standard Weierstrass differentials on E_i defined by $dx/(2y + a_1x + a_3)$.

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matrix_on_subgroup(domain_gens, codomain_gens=None)

Return the matrix by which this isogeny acts on the n-torsion subgroup with respect to the given bases.

INPUT:

- $domain_gens$ basis (P, Q) of some n-torsion subgroup on the domain of this elliptic-curve morphism
- <code>codomain_gens</code> basis (R,S) of the n-torsion on the codomain of this morphism, or (default) <code>None</code> if <code>setf</code> is an endomorphism

OUTPUT:

A 2 × 2 matrix M over \mathbb{Z}/n , such that the image of any point [a]P + [b]Q under this morphism equals [c]R + [d]S where $(c \ d)^T = (a \ b)M$.

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 Code coverage should ideally be 100 %. (This implies we should also test invalid inputs and error cases.)

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- The curve $y^2 = x^3 x + 1$ has 7 points over both \mathbb{F}_3 and \mathbb{F}_9 .

[See https://github.com/sagemath/sage/issues/34467.]

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sage: E .multiplication_by_m_isogeny(-1)
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The .multiplication_by_m_isogeny() method is superseded by .scalar_multiplication().
See https://github.com/sagemath/sage/issues/32826 for details.
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Isogeny of degree 1
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This warning has to remain in place for at least one year before we are allowed to break old code that used to work.

Outline

What is SageMath, really?

The type system The three type systems

Getting your hands dirty

It's working! (That means ε % of the work is done.)

Ready? Go!

- 1. Edit the code in whatever way you think is right.
- 2. Possibly rebuild Sage: make build.
- 3. Check the results!
- 4. Unless satisfied, go back to Step 1 and <u>iterate</u>.

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- 7. Push to a branch on your fork and make a pull request.

 \imath fast path for Vélu isogenies with a single kernel generator imes

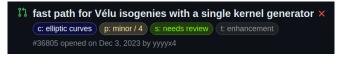
c: elliptic curves p: minor / 4 s: needs review (t: enhancement



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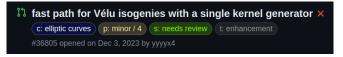
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- Finally (a little while after positive review): It gets merged!
 Congratulations, you're now rich and famous.

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- ► You, too, can be a reviewer! ∵

Hype!!!1!11



https://doc.sagemath.org/html/en/developer