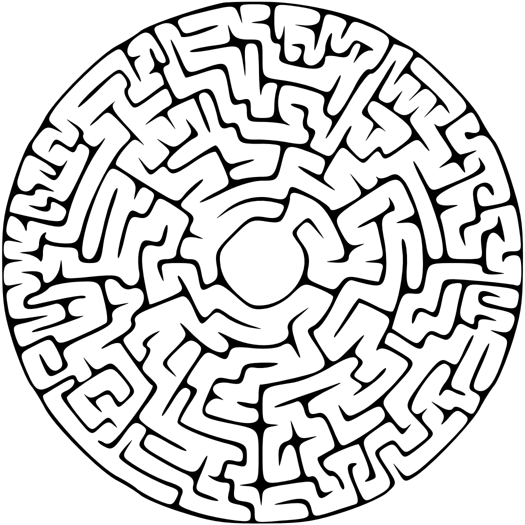
Cryptography on Isogeny Graphs

lekenpraatje

Lorenz Panny

online, 18th February 2021

You're stuck in the center. How do you get out?



You're stuck in the center. How do you get out?

This was easy thanks to our global view.

This was easy thanks to our global view. What about now?

Photo: `https://unsplash.com/@jonminnema`

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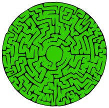

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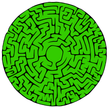

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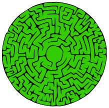

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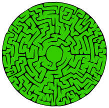

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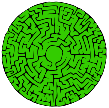

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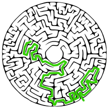

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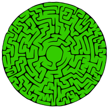

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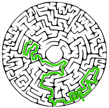

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$\Longrightarrow$ Huge asymmetry in effort!

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Example: (public-key) encryption

- Encrypt using the "easy" random walking.
- Force bad guys to solve the "hard" path finding to decrypt.
- Somehow still enable the recipient to decrypt easily.
  (This is the tricky part.)

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But everyone between them can listen in!

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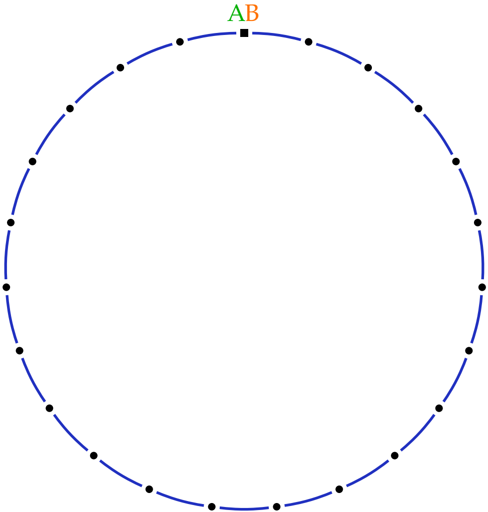
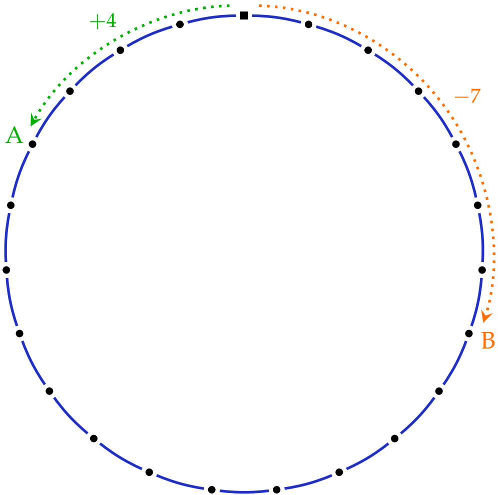
Clearly impossible? No! 🎉

[Diffie & Hellman, 1976]

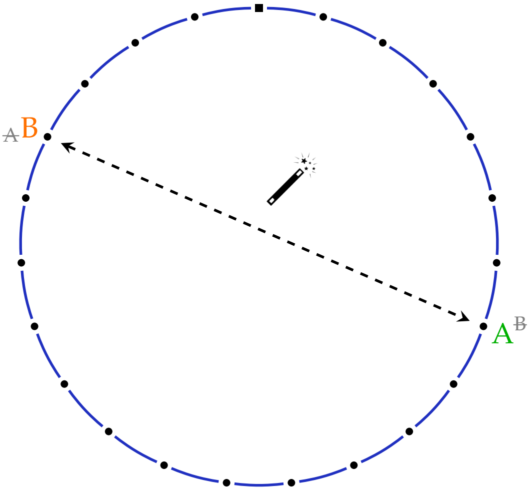
Let’s drop Alice and Bob in this strange-looking “maze”.



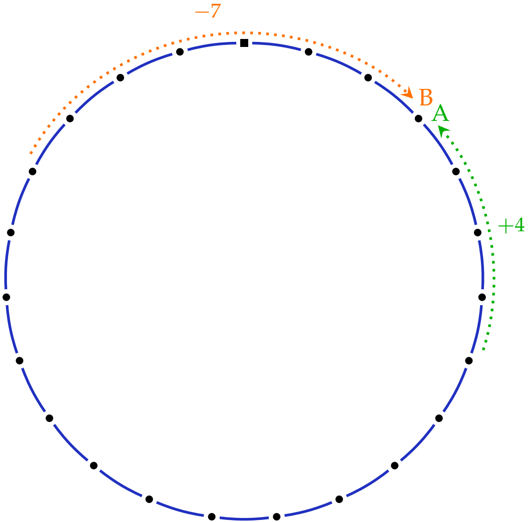
They both pick a random number and walk that many steps.



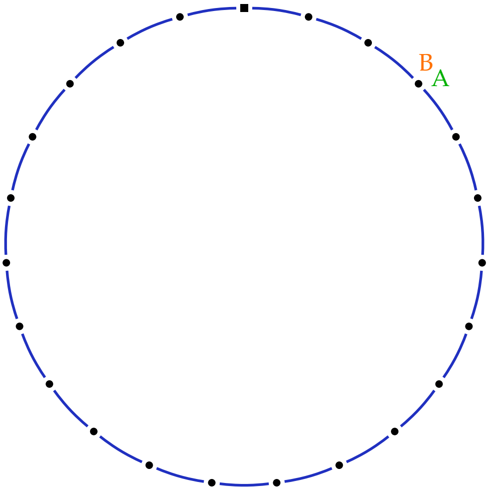
We swap Alice and Bob. (This step requires wizardry.)



They both walk the same number of steps as before.



Alice and Bob arrive at the same location!



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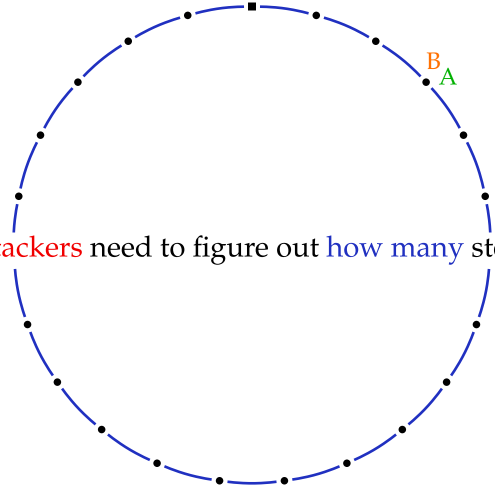
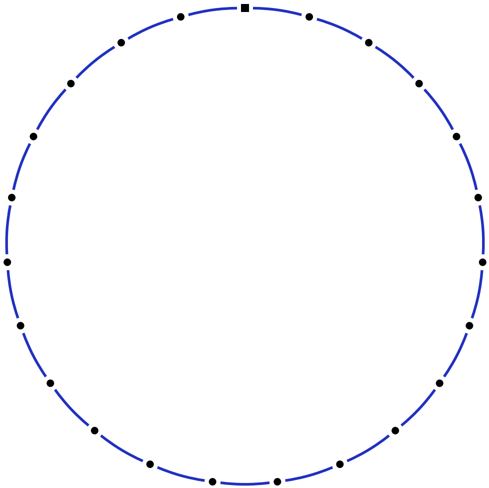


Attackers need to figure out how many steps.

In cryptography, ...

We are doing this in a virtual reality.



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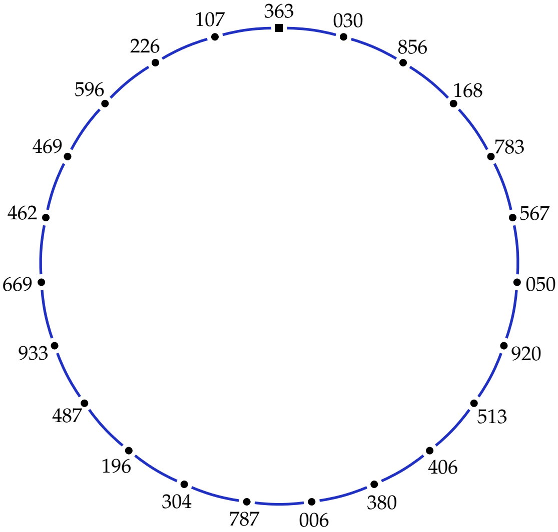
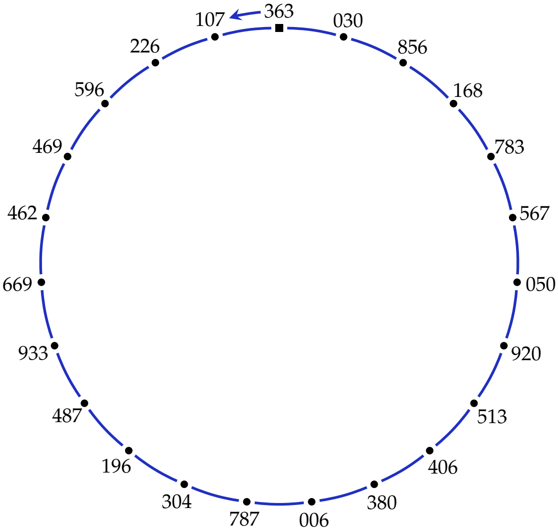
Each location has a *name*.



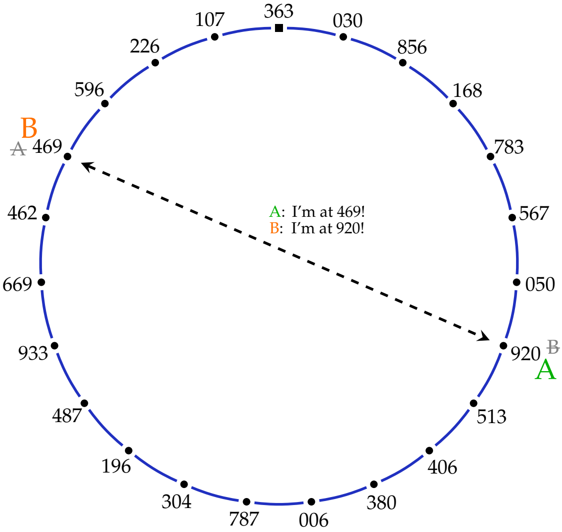
In cryptography, ...

Each step is a *computation*: $(363, \text{left}) \longmapsto 107.$



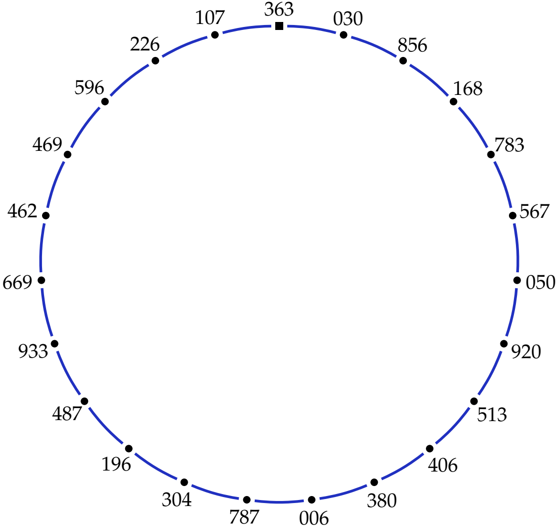
In cryptography, ...

To swap A↔B, we simply exchange place names.



In cryptography, ...

Problem: In this maze, attackers are as fast as Alice and Bob.



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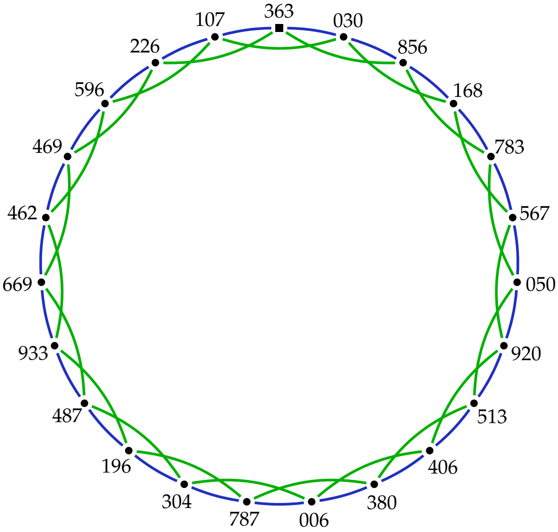

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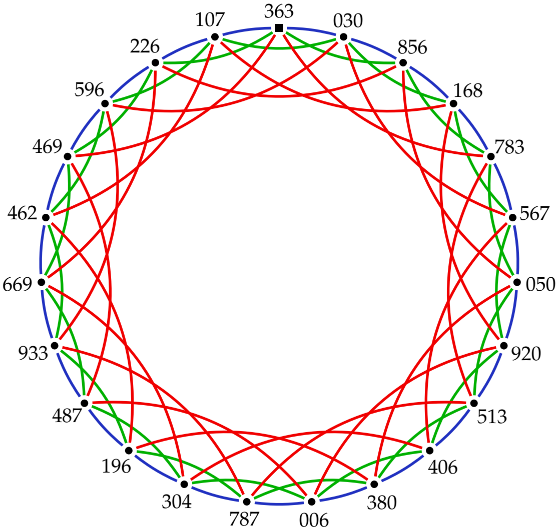

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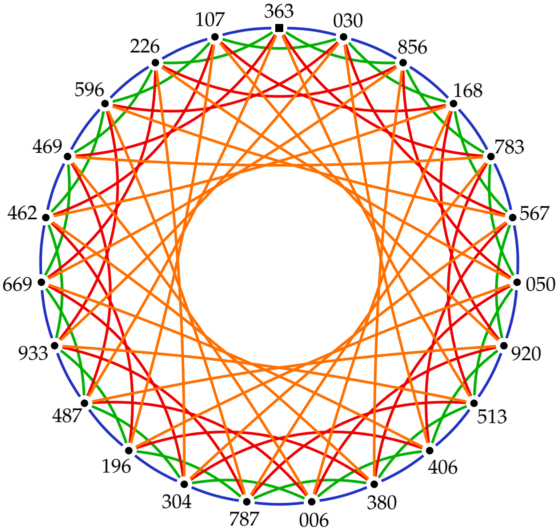

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Photo: `https://unsplash.com/@gferla`

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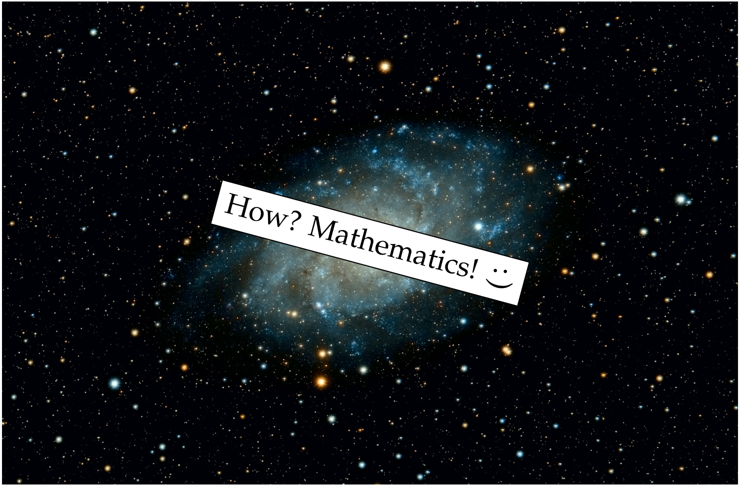

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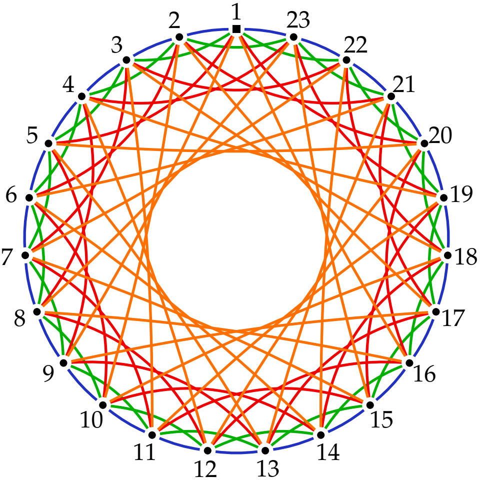
In cryptography, ...

Place *names* must not reveal too much about *where* they are.



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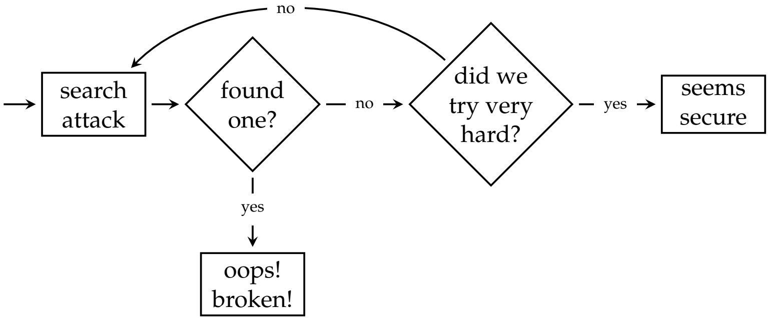
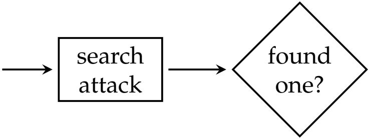
Only method to guarantee this in most cases: *Cryptanalysis*.



The situation for large parts of present-day cryptography:



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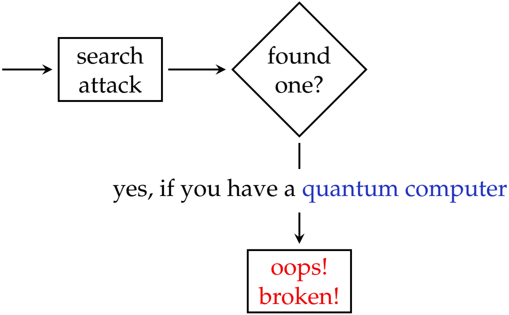

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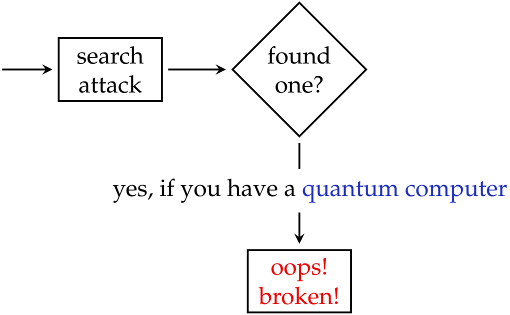

Good news: We are working on ***post-quantum cryptography***!

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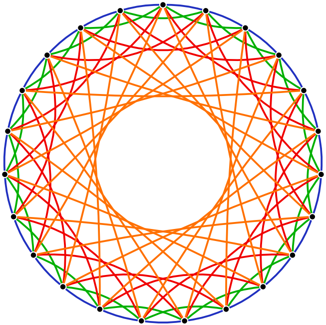

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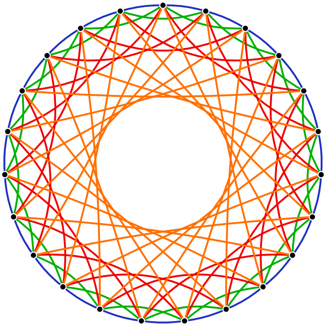

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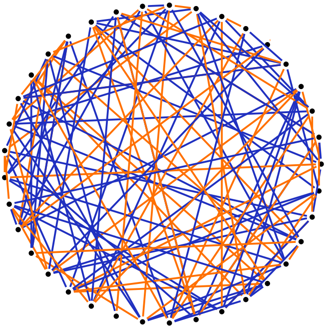
...and this:



(also post-quantum,
and similar math.)

Next up: Questioning!

Bumbling through a conversation with an eminent professor, the grad student outdoes his own stupidity with every remark he makes.

Picture: https://legogradstudent.tumblr.com/post/187145246746