CSIDH:

An Efficient Post-Quantum Commutative Group Action

Wouter Castryck[1] Tanja Lange[2] Chloe Martindale[2]
Lorenz Panny[2] Joost Renes[3]

[1]KU Leuven [2]TU Eindhoven [3]Radboud Universiteit

Hilversum, 20 March 2019

Netherlands Organisation
for Scientific Research

??? math words

I guess it's fast?

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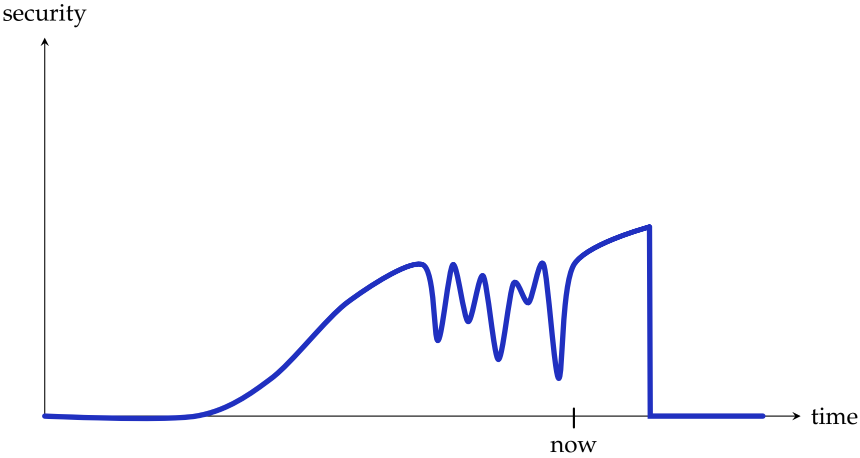
Timeline of internet security (not to scale)



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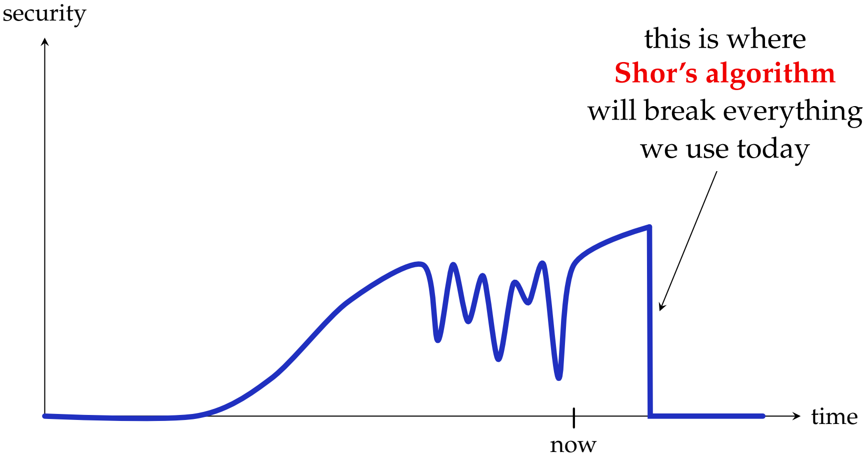
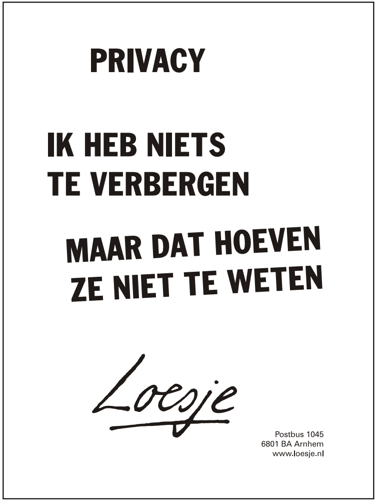


this is where **Shor's algorithm** will break everything we use today



PRIVACY

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TE VERBERGEN

MAAR DAT HOEVEN
ZE NIET TE WETEN

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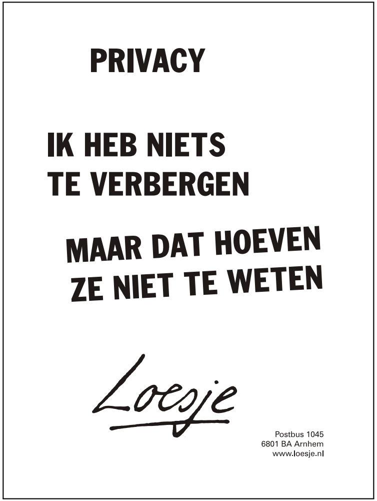
Figure 1: A brief introduction to privacy.

Quantum attacks



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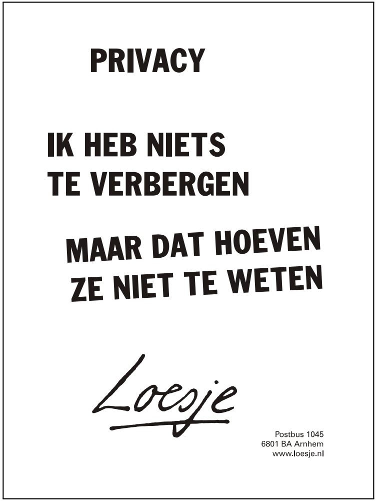

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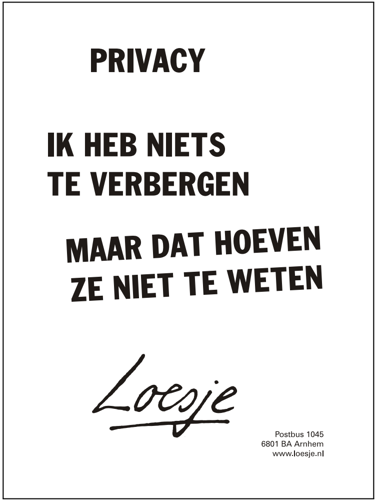

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Figure 1: A brief introduction to privacy.

- Quantum computers will break all common public-key crypto.
- The good news: Nobody has a big enough quantum computer yet.
- The bad news: Attackers run a massive collect-now-decrypt-later effort.
  - Havoc will break loose once they can decipher important secrets...

Shor's algorithm ('94)

Figure 2: Peter W. Shor attacking the crypto in TLS.

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Figure 2: Peter W. Shor attacking the crypto in TLS, and an actual picture of him.

Is all hope lost?

Crypto is probably going to be fine — **if** we act now(-ish).

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  "Quantum computers can do *everything* super fast."
- **Not true!** Many computations have little or no known quantum speedups.

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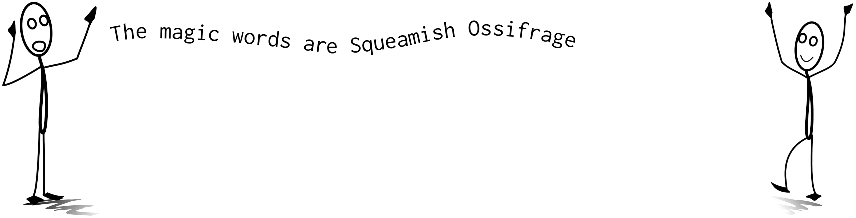
Post-quantum cryptography

uses computational problems where no devastating quantum attacks are known.

The Diffie–Hellman key exchange

Suppose Alice and Bob want to share a secret.



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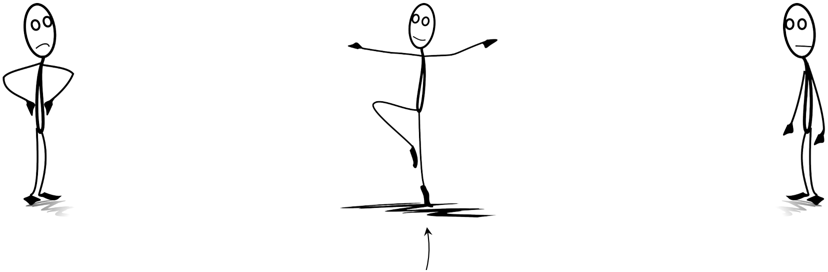

evil eavesdropper Eve!

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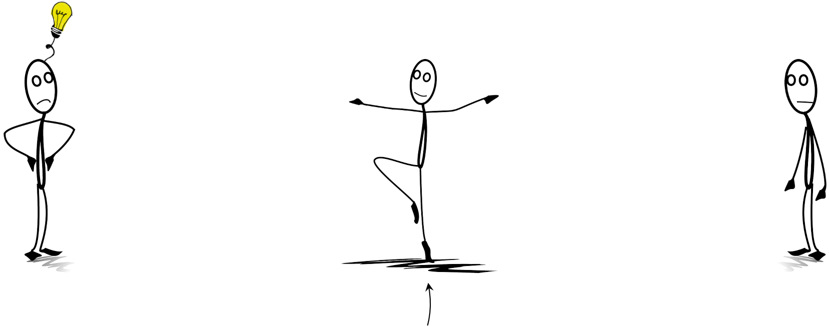

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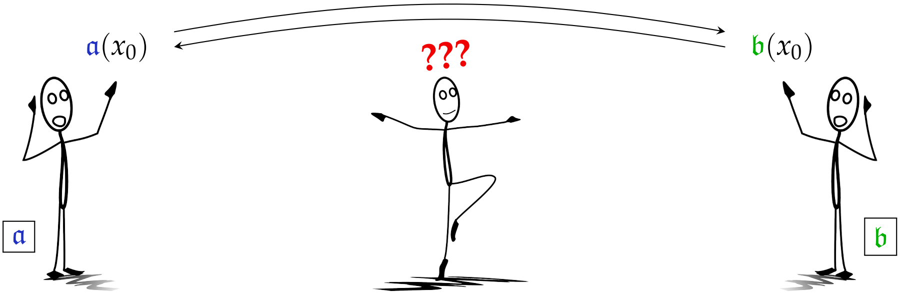

$\mathfrak{a}(x_0)$

???

$\mathfrak{b}(x_0)$

$\boxed{\mathfrak{a}}$

$\boxed{\mathfrak{b}}$

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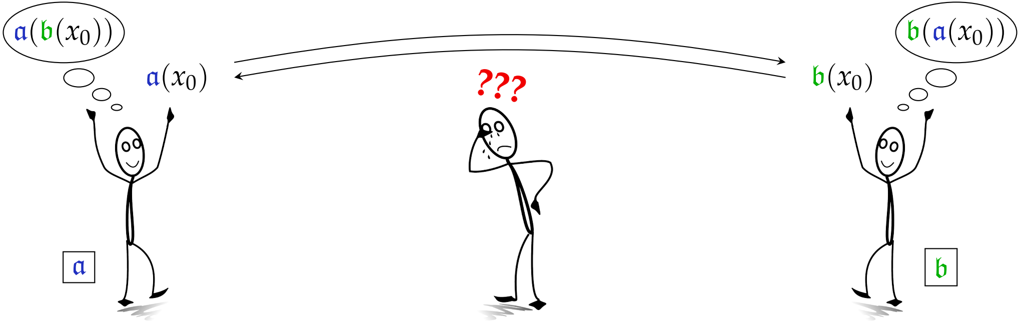

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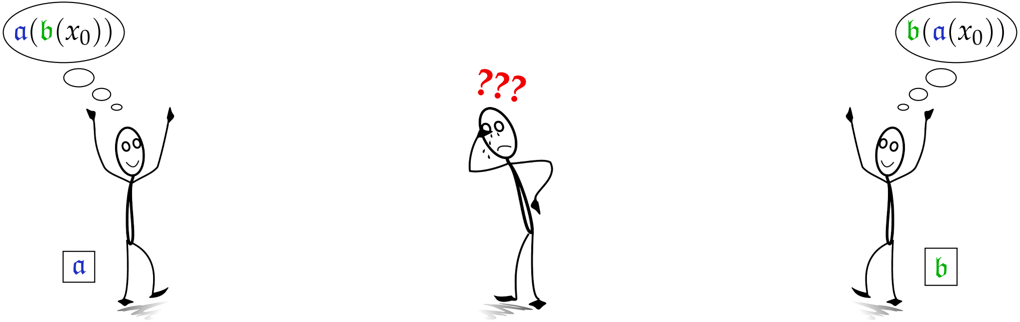
$\mathfrak{a}(\mathfrak{b}(x_0))$
???
$\mathfrak{b}(\mathfrak{a}(x_0))$
$\mathfrak{a}$
$\mathfrak{b}$

- By ~~magic~~ math, $\mathfrak{a}(\mathfrak{b}(x_0)) = \mathfrak{b}(\mathfrak{a}(x_0))$! ...but Eve doesn't know this secret.
- Now Alice and Bob can use their secret to encrypt messages back and forth.

Non-interactive key exchange

The method on the previous slide is an example of a non-interactive key exchange:

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Alice can obtain a *shared secret* by applying her secret $\mathfrak{a}$ to Bob's public key $\mathfrak{b}(x_0)$, and vice-versa. No interaction required after the initial key generation!

Our work: a post-quantum NIKE

Short summary before our work:

All NIKEs either broken by quantum computers or extremely inefficient.

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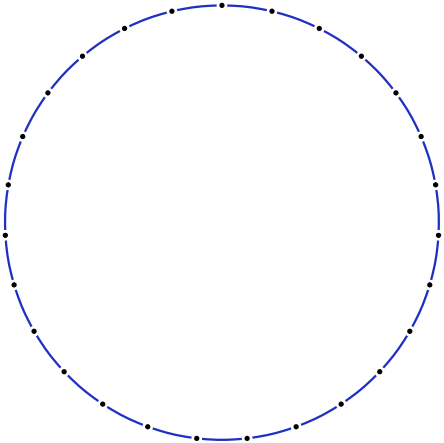
Short summary now:

CSIDH seems post-quantum secure and is reasonably fast!

Our work: the CSIDH graph



(this is just a tiny toy example)

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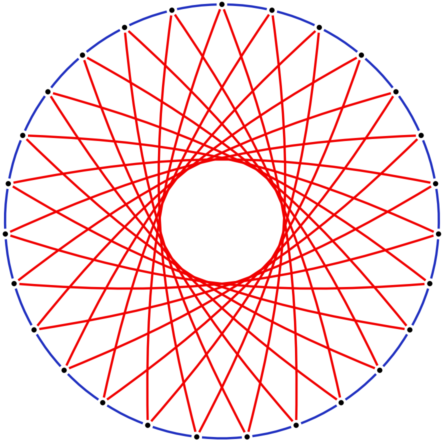

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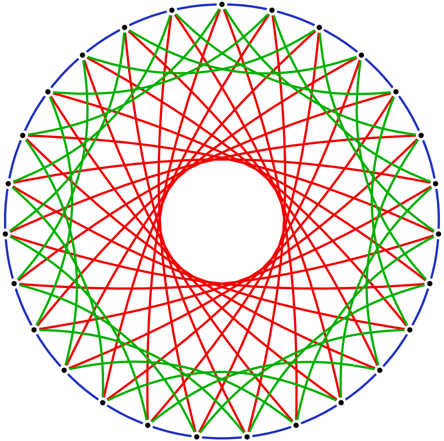

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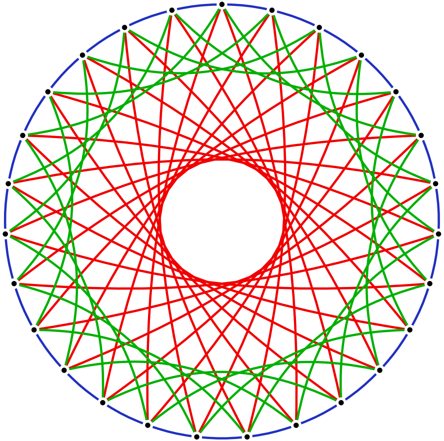

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  right, left, left, left, right, left, right.

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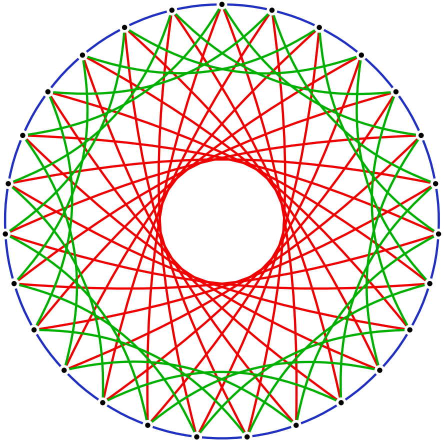

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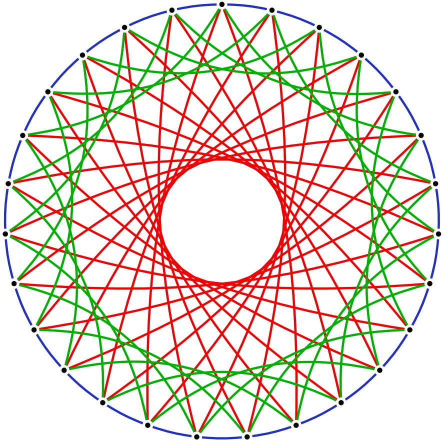

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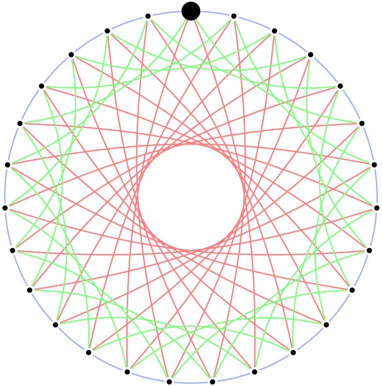
- Alice and Bob can make a key exchange by choosing directions as their secrets $\mathfrak{a}$ and $\mathfrak{b}$ and publishing the end points of walking from a common starting node x_0.

Our work: key exchange on the CSIDH graph

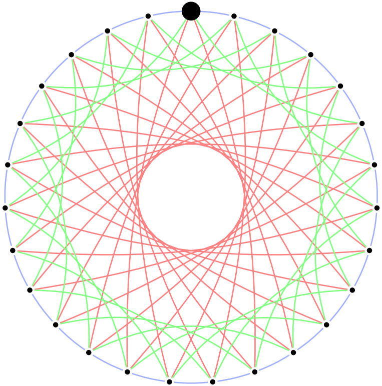
Alice

[+, +, −, −]



Bob

[−, +, −, −]



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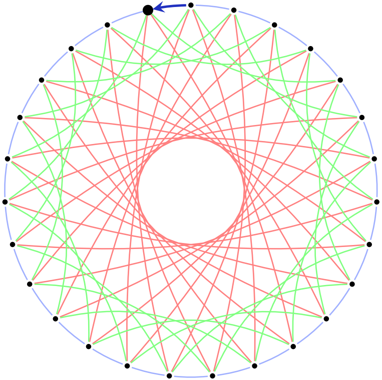

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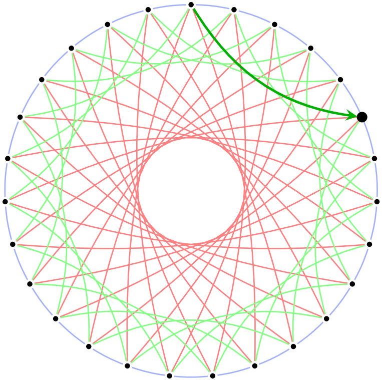

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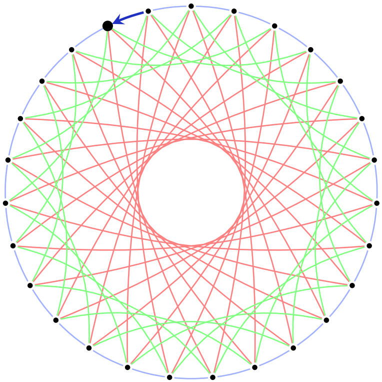

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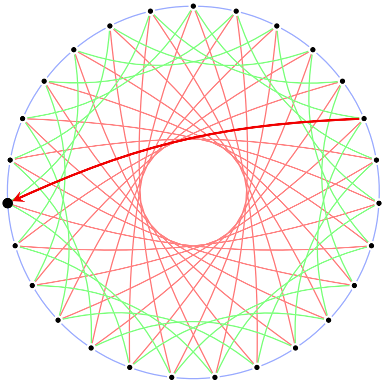

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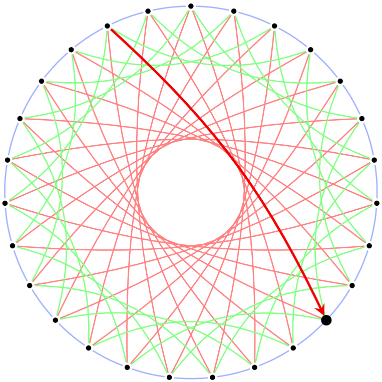

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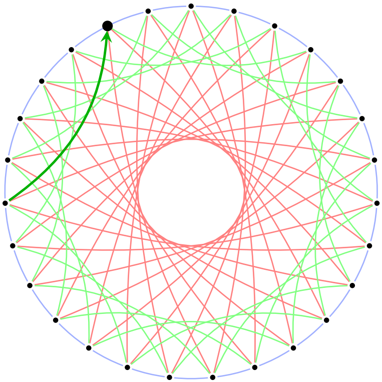

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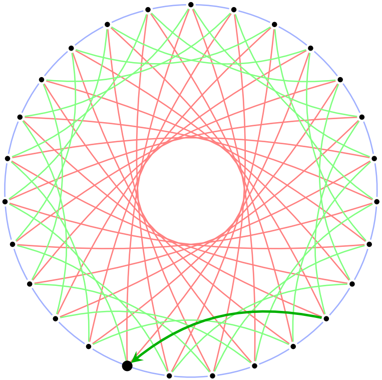

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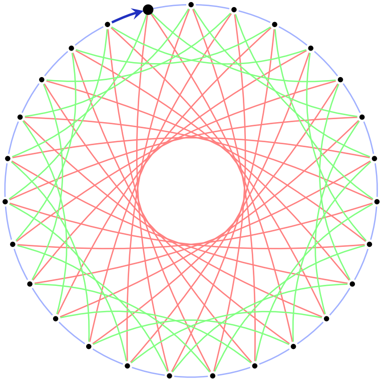

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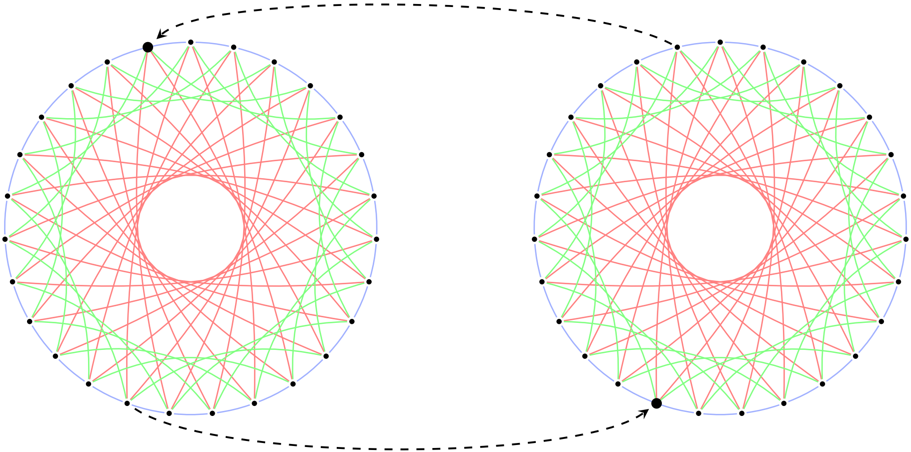

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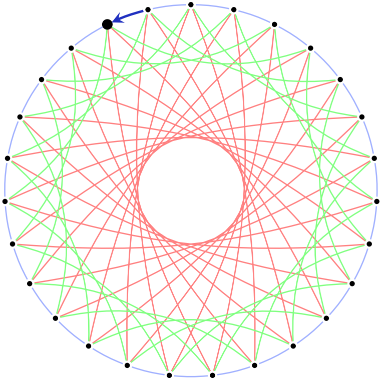

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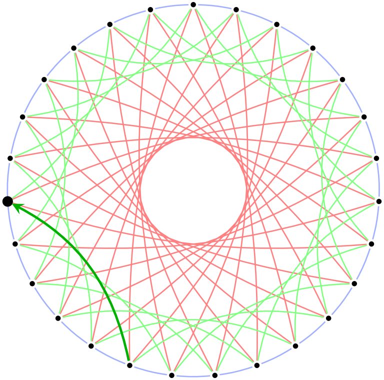

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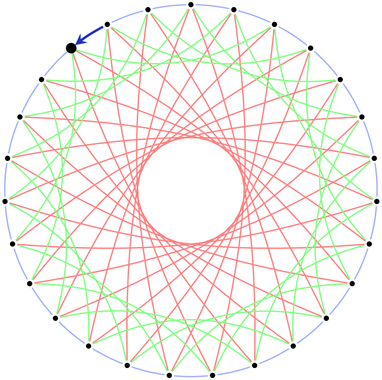

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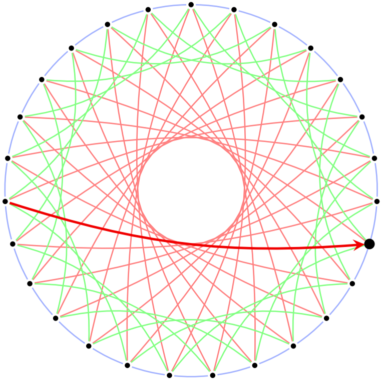

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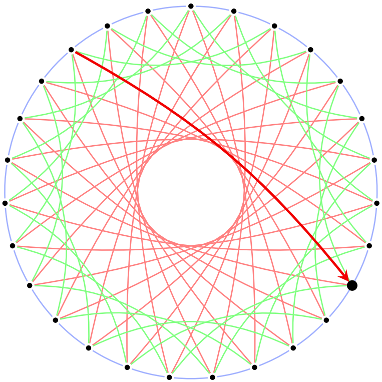

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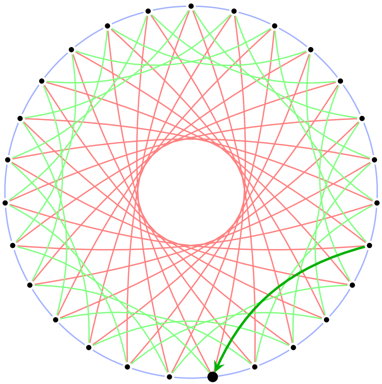

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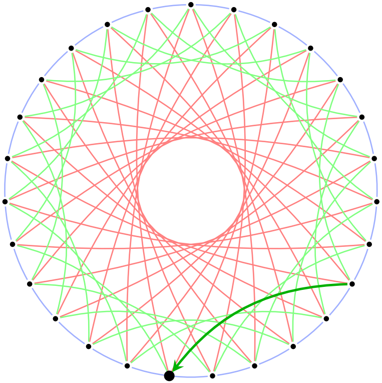

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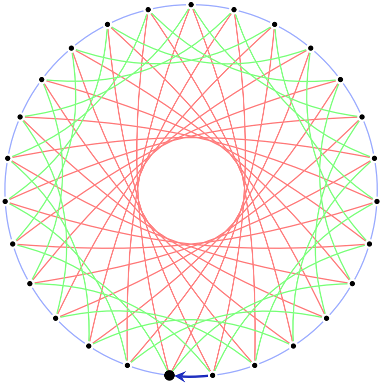

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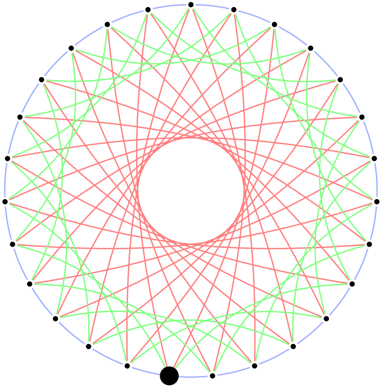

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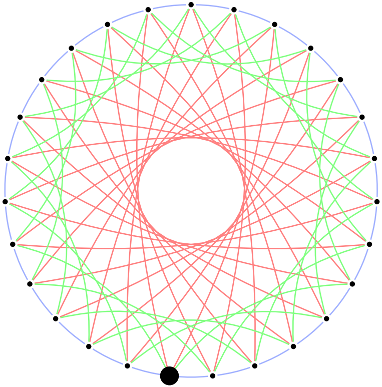

Our work: conclusion

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  $\Longrightarrow$ more flexible security mechanisms for a `cyber future`!

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